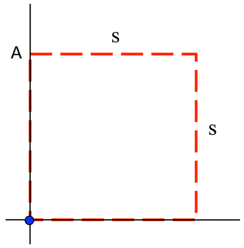
Problem 25.22

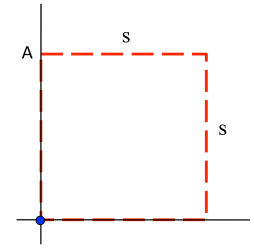


As was the case in Problem 25.14, the amount of energy in a system is equal to the amount of energy you must put into the system to place all the charges at their appointed positions coming in from infinity. We did this with voltages last time. A little different way to look at it is through the idea of potential energy. If we can determine how much potential energy each of the charges has as the system is assembled, we will have our total.

The first charge, place at the origin, is free in the sense that it will take no energy to get it there. As such, its work contribution within the system will be zero.

The amount of work W (where $W = -\Delta U$) done by an electric field on a charge "q" as the charge moves through an electrical potential difference ΔV is:

$$W = -q\Delta V$$



The electrical potential difference experience by a charge moving from infinity (where it's voltage is zero) to a point "s" units from the field producing charge is:

$$\Delta V = \left(k\frac{q}{s} - 0\right) = k\frac{q}{s}$$

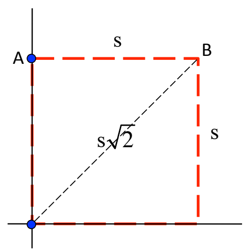
That means the amount of work the field does during that transfer will be:

$$W = -q\left(k\frac{q}{s}\right) = -k\frac{q^2}{s}$$

which means you would have to put energy into the system of the amount

$$k\frac{q^2}{s}$$

to motivate the charge to transfer from infinity to "s." This is the amount of energy in the system once the first two charges are placed.



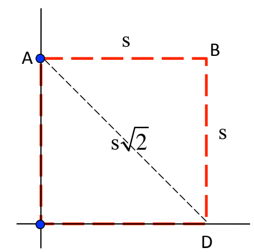
We now have two charges in the system, each contributing to the electrical potential at the next point, which I will take to be the diagonal corner at Point B. One of the charges will be "s" units from B while the other will be $s\sqrt{2}$ from B. That means that along with the original energy, the amount of additional energy you will have to put into the system at that point

With V_2 due to two point charges, it's value will be:

$$k\frac{q}{s\sqrt{2}} + k\frac{q}{s}$$

and the total energy in the system at this point will be:

$$\left(k\frac{q^2}{s}\right) + \left[\left(k\frac{q^2}{s\sqrt{2}}\right) + \left(k\frac{q^2}{s}\right)\right]$$



We now have three charges in the system, each contributing to the electrical potential at the next point, which I will take to be the diagonal corner at Point D. Two of the charges will be "s" units from D while the other will be $s\sqrt{2}$ from D. That means that along with the energy to date, the amount of additional energy you will have to put into the system at that point

$$k\frac{q}{s\sqrt{2}} + 2k\frac{q}{s}$$

and the total energy in the system ends up at:

$$\left(k\frac{q^2}{s}\right) + \left[\left(k\frac{q^2}{s\sqrt{2}}\right) + \left(k\frac{q^2}{s}\right)\right] + \left[\left(k\frac{q^2}{s\sqrt{2}}\right) + 2\left(k\frac{q^2}{s}\right)\right]$$

$$= 4k\frac{q^2}{s} + 2k\frac{q^2}{s\sqrt{2}}$$

$$= k\frac{q^2}{s}\left(4 + \frac{1}{\sqrt{2}}\right)$$

$$= 5.41k\frac{q^2}{s}$$

NASTY, EH?